RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIFTH SEMESTER EXAMINATION, DECEMBER 2019

THIRD YEAR [BATCH 2017-20]

Date : 14/12/2019 **PHYSICS (Honours)**

Time : 11 am – 1 pm **Paper : VI [Gr. A]** Full Marks : 50

Answer **any five** questions of the following: [5×10]

1. Discuss briefly how the concept of wave-particle duality for radiation was established by analysing the following:
   a) Black-body radiation
   b) Photo-electric effect
   c) Compton scattering

   What are the mathematical expressions of this wave-particle duality? (3+3+3+1)

2. a) What is De Broglie's concept of ***matter wave***?
   Obtain an expression for the De Broglie wavelength of a particle of mass μ and speed *v*. (6)

   b) What is the wavelength associated with an electron having energy $E = 200\,eV$? (4)
   [$m_e = 9\times10^{-31}\,kg$ and $h = 6.62\times10^{-34}\,m^2\text{-}kg/s$]

3. a) Distinguish between the group velocity v_g and phase velocity v_p of a wave packet.
   How are they related?
   Show that for a wave-packet of a non-relativistic particle with velocity $v = v_g$ (6)

   b) State and explain clearly Heisenberg's uncertainty principle.
   How is it related to the wave-particle duality of matter? (4)

4. a) Why are observables in *QM* represented by given Hermitian operators?
   Write down the operator representation for i) Position, ii) linear momentum and iii) energy in one dimension. Hence show that the commutator $\left[x, p\right] = i\hbar$. (2+4)

   b) Show that if two quantum-mechanical operator commute they have the same set of Eigen functions. (4)

5. a) Write down the **time-dependent schrödinger equation** (*TDSE*) for a particle moving in a One-dimension in potential field $V(x)$.
   Hence obtain the time-independent of the wave equation. (4+1)

   b) If the probability density $p(x,t)$ to find the particle at point *x* and time *t* is given by $p(x_1 t) = |\psi(x,t)|^2$, where $\psi(x,t)$ satisfies the *TDSE*. Then show that $p(x,t)$ satisfies One-dimensional continuity equation. (5)

6. Determine the energy levels and corresponding normalised Eigen function of a particle in a one dimensional potential as follows:

$$V(x) = \infty \quad for\ x < 0\ and\ x > a$$
$$= 0 \quad for\ 0 \le x \le a$$
(10)

7. Consider $V(x) = \frac{1}{2}kx^2$ is the potential energy of linear harmonic oscillator. Prove that energy Eigen values are $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$. Find the ground-state wave function. (10)

8. a) Write the three-dimensional time-independent Schrodinger equation for a particle moving in a central potential $V(\text{r})$ and express it in spherical polar coordinates (r, θ, ϕ). (5)

   b) Use the method of separation of variables to obtain the differential equations for the radial and angular wave functions. (5)

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